Effects of Fiber Length on the Tensile Strength of Epoxy/Glass Fiber and Polyester/Glass Fiber Composites

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Synopsis

In discontinuous fiber-reinforced composites, the shear strength at the fiber–matrix interface plays an important role in determining the reinforcing effect. In this paper, a method was devised to accurately determine this shear strength, taking the strength distribution of glass fiber into consideration. Calculated strength values based on the shear strength obtained by the method were in better agreement with the experimental observations than those calculated by employing the shear strength obtained on the assumption that the fiber strength was uniform. The tensile strength of composites increases with increasing aspect ratio of the reinforcing fibers. This trend is almost the same regardless of the kind of matrix, the nature of interfacial treatment, and the environmental temperature. When composites are reinforced with random-planar orientation of short glass fibers of 1.5 times the mean critical fiber length, the tensile strength of composite reaches about 90% of the theoretical strength of composites reinforced with continuous glass fiber. Reinforcing with glass fibers 5 times the critical length, the tensile strength reaches about 97% of theoretical. However, from a practical point of view, it is adequate to reinforce with short fibers of 1.5–2.0 times the mean critical fiber length.

INTRODUCTION

As is well known, loads working on the short-fiber-reinforced composites are transmitted to the fiber through shear at the fiber–matrix interface. Consequently, the strength of the composite is greatly influenced by the shear strength at the interface by the critical fiber length, which is dependent on the shear strength. Hence, in discontinuous fiber-reinforced resins, precise determination of critical fiber length is of great importance. However, the conventional experimental methods[1–3] are indirect methods and include various errors and do not give good reproducibility. In the preceding paper,[4] we reported a method for directly measuring critical fiber length. According to this method, we clearly explained the temperature dependence of critical aspect ratio and discussed theoretically and experimentally the temperature dependence of tensile strength and that of impact fracture energy using the results obtained by the method.[5,6]

The strength of glass fibers and carbon fibers is variable with fiber length, however, in the studies reporting on the strength of the composites reinforced with those fibers, including our preceding article,[5] the strength of fibers is assumed to be logically and experimentally uniform, and the effect of the strength variation is not considered.[7–9] Only Fraser et al.[10] proposed a method for estimating the shear strength at the interface taking into account the effect of variation of fiber strength, yet the relation between the shear strength at the

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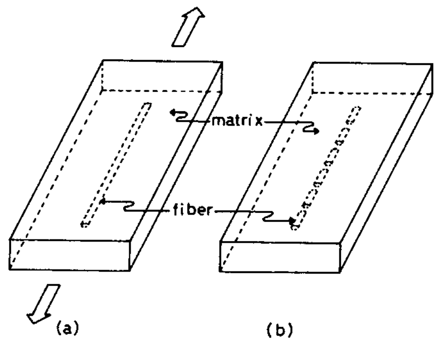

Fig. 1. Schematic representation of model used in measuring fragment length.

interface and strength of the composite is not discussed. The effect of fiber length on the strength of composites reinforced with unidirectionally oriented short fibers has been studied; however, there is practically no study for composites in which the short fibers are randomly oriented two- or three-dimensionally.

In this paper, efforts were made to determine accurately the shear strength at the interface or the critical fiber length taking the variation of fiber strength into consideration. Furthermore, the effect of fiber length on the tensile strength of composites was investigated, and the shear strength and the critical fiber length were applied to the composite systems.

MEASUREMENTS OF CRITICAL FIBER LENGTH

Principle

As described in the preceding paper,[4] if a sufficiently long fiber is embedded in the resin matrix and the system is elongated, Figure 1(a), the fiber eventually breaks into many pieces, Figure 1(b). Measuring the lengths of the broken pieces, the critical fiber length for the system can be estimated. Assuming that the fiber strength is distributive, the critical fiber length and the shear strength at the interface are estimated as follows.

The tensile strength of brittle fibers such as glass fiber is greatly affected by partial flaws. In general, the tensile strength of such fibers is represented by a chain model (Fig. 2). This model represents a fiber by a chain constituted of n pieces of equal links. The strength of each link is dependent on flaws existing within the link so that the strength of the chain is determined by the strength of a link having the most intensive flaw. Let $f(\sigma)$ represent the probability density function (PDF) characterizing the strength of the links, $F(\sigma)$ the associated cumulative distribution function, and $g(\sigma)$ the PDF characterizing the total fiber or chain strength. Thus, the probability $g(\sigma)$ that a chain of n links will break at a stress σ is equal to the probability $f(\sigma)$ that one link will fail at a

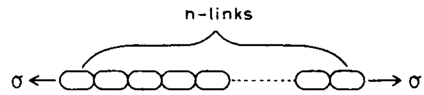

Fig. 2. Schematic representation of assumed chain model of fiber.

stress σ, times the probability $[1 - F(\sigma)]^{n-1}$ that the $n - 1$ links are still serving, times the number of combinations, i.e., the number of links n, which can be expressed as

$$g(\sigma) = n\, f(\sigma)\, [1 - F(\sigma)]^{n-1} \tag{1}$$

Applying the Weibull distribution function,[12] eq. (1) can be expressed as

$$g(\sigma) = nm\sigma_0^{-1} \left(\frac{\sigma - \sigma_p}{\sigma_0}\right)^{m-1} \exp\left\{-n \left(\frac{\sigma - \sigma_p}{\sigma_0}\right)^m\right\} \tag{2}$$

where σ_0, σ_p, and m are the Weibull parameters determined for the material, σ_0 and σ_p being the maximum and the minimum values of the tensile strength of the fiber, respectively, obtained by the tensile strength test of many fibers of a fixed gauge length. The parameter m and the number of links n are selectively determined so that closest agreement between experimental and theoretical values is obtained.

Accordingly, the mean tensile strength of the fiber $\bar{\sigma}_f$ is given as

$$\bar{\sigma}_f = \int_0^\infty \sigma\, g(\sigma)\, d\sigma \tag{3}$$

which on integration yields

$$\bar{\sigma}_f = \sigma_p + (\sigma_0/n^{1/m})\, \Gamma\left(\frac{m+1}{m}\right) \tag{4}$$

where Γ is the complete gamma function. On substitution of $n = 1$ into eq. (2), we obtain eq. (5) for the probability density function $f(\sigma)$:

$$f(\sigma) = m\sigma_0^{-1} \left(\frac{\sigma - \sigma_p}{\sigma_0}\right)^{m-1} \exp\left\{-\left(\frac{\sigma - \sigma_p}{\sigma_0}\right)^m\right\} \tag{5}$$

When the final fragment length, Figure 1(b), is shorter compared with the length of the links constituting the fiber, the fragment length can be estimated as follows. In such a case the fragment length is equivalent to the final length of the fragments, when the links of the chain model of Figure 2 independently are embedded in the resin matrix as shown in Figure 1(a), and the system is elongated. As the strength distribution is uniform along the length of each link, the method established in the preceding paper[4] based upon the assumption that the fiber strength is uniform along the length of the fiber can be used. Accordingly, the mean fragment length $\bar{l}_l$ for each link is given by:

$$\bar{l}_l = \frac{3}{4} l_{cl} \tag{6}$$

where l_{cl} is the critical fiber length in each link.

According to Kelly et al.,[1,13] while assuming that the yield shear stress (τ) at the fiber–matrix interface is constant, the relation between the critical fiber length l_{cl} and the fiber strength results in the following equation:

$$l_{cl} = \frac{d}{2\tau} \sigma_{fl} \tag{7}$$

where σ_{fl} is the strength of each link and d is the diameter of the fiber. On substitution of eq. (6) into eq. (7), we obtain

$$\bar{l}_l = \frac{3d}{8\tau} \sigma_{fl} \tag{8}$$

Then the fragment length distribution $f(\bar{l}_l)$ obtained over the whole length of the fiber constituted of these links is obtained on substitution of $f(\sigma)$ given by eq. (5) into σ_{fl} given by eq. (8):

$$f(\bar{l}_l) = \frac{3d}{8\tau} f(\sigma) = \frac{3d}{8\tau} m \sigma_0^{-1} \left(\frac{\sigma - \sigma_p}{\sigma_0}\right)^{m-1} \exp\left\{-\left(\frac{\sigma - \sigma_p}{\sigma_0}\right)^m\right\} \tag{9}$$

The above equation shows that the fragment length distribution depends in a measure upon the shear strength τ at the fiber–matrix interface in addition to the statistic factors σ_0, σ_p, and m of the fiber strength. Accordingly, the shear strength τ at the interface for a system comprising fibers having a distributive strength is determined as follows.

First, the fragment length distribution of the system is measured according to the method provided in our preceding paper.[4] Second, the statistic values of factors σ_0, σ_p, and m of the fiber strength used in the system are determined. Substituting these values into eq. (9), the value of τ is selectively determined so that $f(\bar{l}_l)$ may best agree with the measured fragment length distribution. The value of τ thus determined represents the shear strength at the fiber–matrix interface.

Mean critical fiber length $\bar{l}_c$ is given by

$$\bar{l}_c = \frac{\bar{\sigma}_{fl} d}{2\tau} \tag{10}$$

where $\bar{\sigma}_{fl}$ is the mean value of strength of the links which is shown on substitution of $n = 1$ into eq. (4) as:

$$\bar{\sigma}_{fl} = \sigma_p + \sigma_0 \Gamma \left(\frac{m+1}{m}\right) \tag{11}$$

Preparation of Specimens

The glass fibers and the resins used in preparation of the specimens are the same as those described in the preceding report.[4–6] Namely, the fiber used is "E" glass fiber (R 2220 MA859 XL16, 12.73 μm in diameter, Asahi Fiber Glass), and the matrix materials are an epoxy resin (Epikote 828, Mitsubishi Yuka) and an unsaturated polyester resin (Rigolac 2004 WM-2, Showa Kobunshi).

First, tensile tests were performed using fibers of 3- and 10-cm gauge length to obtain strength distribution curves for the glass fiber. The instrument used was Tensilon UTM-III type (Toyo Baldwin), strain rate 0.05 mm/mm/min, and the number of fibers tested was 200 pieces for each gauge length. The results obtained by the fiber strength distribution tests were used to determine the number of links n of eq. (2) and the Weibull parameters m, σ_0, and σ_p as shown in Table I.

Second, specimens were prepared for measuring fragment length under the same conditions as reported in the preceding paper.[4] The glass fiber was boiled in distilled water for about 10 min and dried at 80°C for 8 hr. To prepare specimens differing in bond strength at the interface, the glass fiber was immersed in 3% toluene solution of a silane coupling agent (KBM403, Shinetsu Chemical) or in a 6% toluene solution of a release agent (KS707, Shinetsu Chemical) for 48 hr, air dried, and then dried at 70°C for 8 hr.

TABLE I
Statistical Values of Strength for Glass Fibers

Test length (cm)	σ_f ($\times 10^{10}$ dyn/cm^2)	σ_0 ($\times 10^{10}$ dyn/cm^2)	σ_p ($\times 10^{10}$ dyn/cm^2)	m	Number of links n
3.0	2.45				5.95
10.0	2.09	3.71	0.30	4.3	20.10
Link	3.68				1.00

TABLE II
Temperatures Used in Measuring Fragment Length

Matrix	Temperature, °C			
Epoxy resin	40	60	80	100
Unsaturated polyester resin	50	60	80	100

Epoxy resin, 100 parts, was mixed with 10 parts amine hardening agent (S-Cure 661, Nihon Kayaku), or 100 parts unsaturated polyester resin was mixed with 2 parts hardening catalyst MEKPO (Permek N, Nihon Yushi). The mixture was agitated thoroughly and then deformed under vaccum for about 20 min. This mixture was poured into a mold holding a glass fiber in the center and subjected to curing at 65°C for 17 hr and postcuring at 140°C for 5 hr. The specimen was then allowed to cool to room temperature at a cooling rate of about 0.5°C/min. The dimension of the mold resulted in a specimen which measured 0.5 mm in thickness, 20 mm in width, and 150 mm in length.

The specimens prepared in this manner were submitted to measurement of the fragment length distribution: each specimen was subjected to a tensile strain greater than the fiber ultimate tensile strain at a test rate of 0.05 mm/mm/min at various temperatures (Table II). Over 100 fragments were used for each experimental condition.

Results

The strength distributions of glass fibers used in the experiments is shown in Figure 3. The solid lines in the figure represent theoretical values obtained

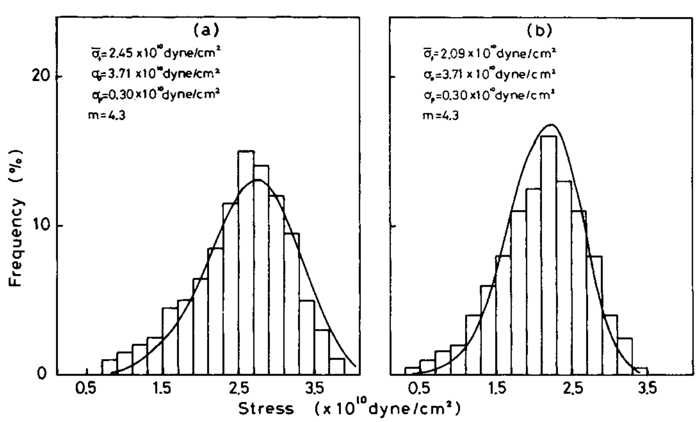

Fig. 3. Strength distribution of glass fiber: (a) test length 3 cm; (b) test length 10 cm.

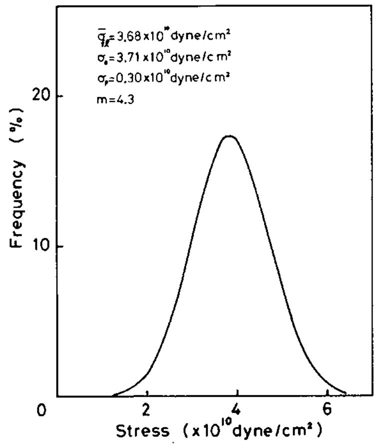

Fig. 4. Strength distribution function of assumed glass fiber link.

by substituting Weibull parameters σ_0, σ_p, m, and the number of links n given (Table I) into eq. (2). The length of the link (gauge length/number of links) is 5.00 mm. Calculation of the link strength distribution function substituting the Weibull parameters (Table I) into eq. (5) results in a theoretical curve as shown in Figure 4 where mean strength $\bar{\sigma}_{fl}$ is 3.68 × 10^{10} dyne/cm^2.

Figure 5 illustrates a typical fragment length distribution of a composite system including glass fiber having a strength distribution as described above. Apparently, all fragments are shorter than the link length of the glass fiber, 5 mm. According to the measurement theory, a shear strength τ at the interface that will give a distribution curve which best agrees with measured fragment length distribution curve was determined by substituting the Weibull parameters σ_0, σ_p, and m (Table I) into eq. (9). The theoretical distribution curve (the solid lines in Fig. 5) obtained by eq. (9) employing the value of τ well agrees with the measured distribution curve. Therefore, the value of τ thus determined is appropriate.

The described results represent characteristics of the system of epoxy resin and glass fiber treated with the coupling agent. Fragment length is shorter than

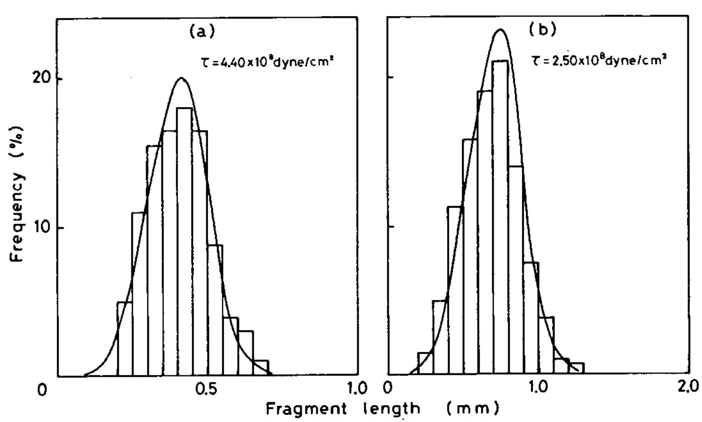

Fig. 5. Distribution of fragment length for the epoxy–glass fiber system (good bonding): (a) at 40°C; (b) at 80°C.

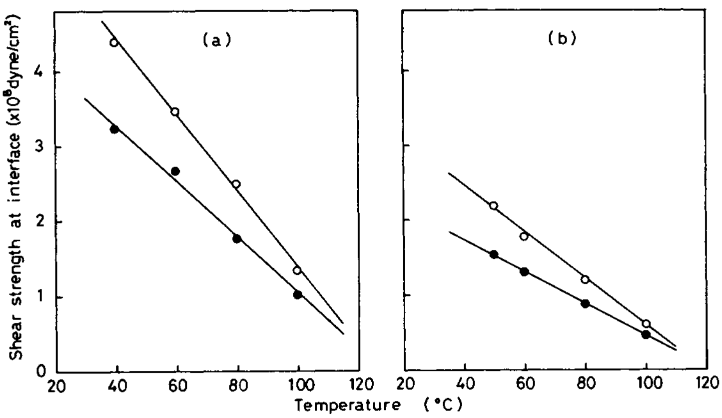

Fig. 6. Relation between temperature and shear strength at interface: (a) epoxy–glass fiber interface; (b) unsaturated polyester–glass fiber interface; (—O—) good bonding; (—●—) poor bonding.

the link length also with the unsaturated polyester system or with any interfacial treatment and at any environmental temperature. Therefore, a similar method was available for determining shear strength τ at the interface for every condition.

The relationship between the shear strength τ at the interface and the temperature is shown for the epoxy–glass fiber and unsaturated polyester–glass fiber systems in Figure 6, taking the strength distribution of the fiber into consideration. With these systems, regardless of the nature of interfacial treatment, the shear strength at the interface decreases almost linearly with increasing temperature as reported before.[4]

The relationship between the mean critical aspect ratio $\bar{l}_c/d$ obtained from the shear strength at the interface according to eq. (10) and the temperature is shown in Figure 7. Regardless of the nature of interfacial treatment, the mean critical aspect ratio $\bar{l}_c/d$ increases greatly with increasing temperature.

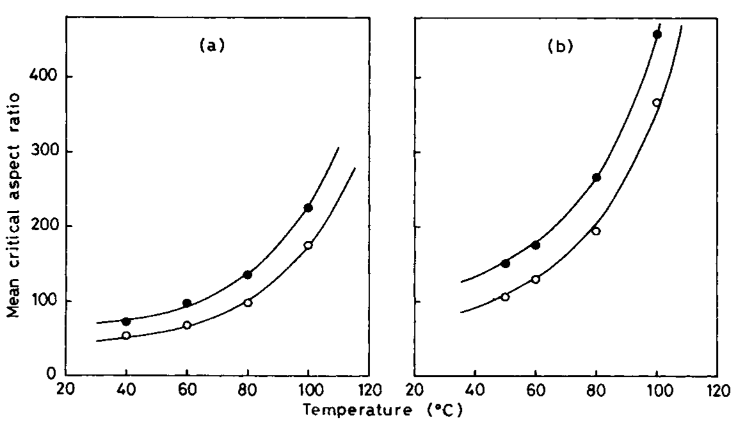

Fig. 7. Relation between temperature and mean critical aspect ratio: (a) epoxy–glass fiber system; (b) unsaturated polyester–glass fiber system; (—O—) good bonding; (—●—) poor bonding.

EFFECTS OF FIBER LENGTH ON THE STRENGTH OF COMPOSITE MATERIALS

Experimental

The glass fibers and resins used in preparation of the specimens are the same as those used in the previously performed critical fiber length measuring experiment and the same as those described in the preceding paper.[4–6] The composites reinforced with random-planar orientation of short fibers were prepared as reported before.[5,6]

It is known that the reinforcement effect is greatly affected by the length of the fiber used as the reinforcements relative to the critical fiber length of the system. In this experiment, four fiber lengths in the range of 0.75 to 5.00 mm were used as reinforcement. In order to make a close study on the effects of the fiber length on the strength of fiber-reinforced composites, special care was exercised to make the length of fibers uniform for every fiber length and to handle the fibers so as not to break them.

Fibers in roving form were bundled with the use of a 4% aqueous solution of PVA, dried for two days, and cut into short fibers of uniform length by a constant-length cutter. After cutting, the PVA was dissolved and removed in water and the short fibers were suspended in a large amount of distilled water and then allowed to settle gently on a filter paper placed at the bottom of the vessel. The water was removed by pressing at a suitable pressure. The fibers were then dried at 80°C for 24 hr. In this manner it was possible to obtain a mat in which the short fibers were oriented random-planarly and yet distributed uniformly. The mean fiber length $\bar{L}$ is shown in Table III.

To prepare specimens differing in bonding strength at the interface, the random mats were treated with the coupling agent or the release agent in the same procedure as in our preceding paper[5,6] and in the above experiments on the critical fiber length. The resin mixtures were also prepared and defoamed in a similar method as in our preceding papers,[4–6] and the above experiments on the critical fiber length.

The random mats were then evacuated thoroughly to remove the air entrapped in the fiber mats, and the resin mixture was poured into the apparatus so that mats could be impregnated fully. Thereafter, atmospheric pressure was gradually applied to expedite the resin mixture. Finally, the impregnated mats were hardened and then cooled down to room temperature, in similar conditions as in our preceding papers[4–6] and in the above experiments.

This procedure enables preparation of bubble-free resins reinforced with random-planar orientation of short fibers. The volume fraction of the glass fibers can be controlled by pressing that mat before impregnation. In this experiment, the volume fraction was set at 10.2% for the epoxy resin composite and at 10.1% for the unsaturated polyester resin composite.

TABLE III
Mean Fiber Length

	No. I	II	III	IV
$\bar{L}$, mm	0.76	1.10	2.69	5.07

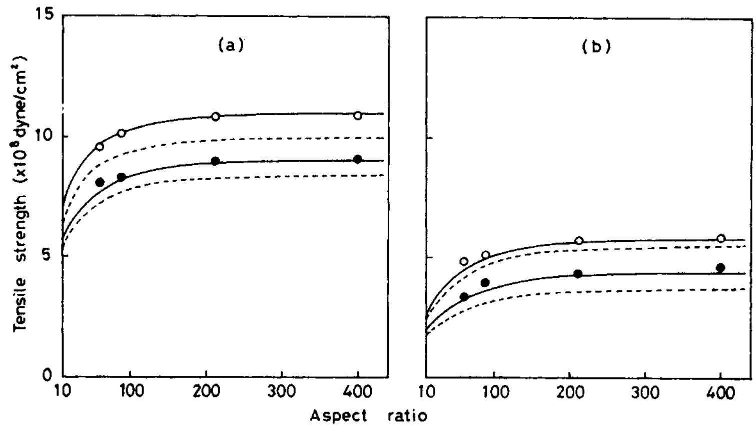

Fig. 8. Relation between aspect ratio and tensile strength (at 40°C): (a) epoxy–short glass fiber composite; (b) unsaturated polyester–short glass fiber composite; (O) good bonding; (●) poor bonding; (—) calculated value from eq. (14) (fiber strength is distributive); (- - - -) calculated value from eq. (14) (fiber strength is uniform).

The test specimens were cut from these composites in accordance with JIS 7113 and subjected to tensile tests at a test speed of 0.15 mm/mm/min with the aid of a Tensilon UTM-I-2500 (Toyo Baldwin). The measuring temperatures are selectively determined at 40 and 80°C considering the results of the above experiments on critical fiber length. Seven to ten specimens were used for each temperature level tested.

Results and Discussion

The relationship between tensile strength and aspect ratio (L/d) for the epoxy–short glass fiber composite and unsaturated polyester–short glass fiber composite at 40°C is shown in Figure 8, while that at 80°C is shown in Figure 9. With these composites, regardless of the nature of interfacial treatment and the

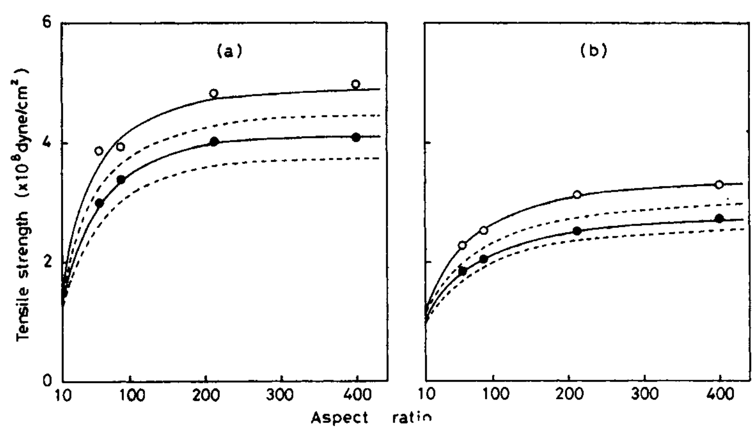

Fig. 9. Relation between aspect ratio and tensile strength (at 80°C): (a) epoxy–short glass fiber composite; (b) unsaturated polyester–short glass fiber composite; (O) good bonding; (●) poor bonding; (—) calculated value from eq. (14) (fiber strength is distributive); (- - - -) calculated value from eq. (14) (fiber strength is uniform).

atmospheric temperature, the tensile strength increases rapidly as the aspect ratio increases; however, the increase in tensile strength has remained almost unchanged after the aspect ratio has reached a certain level.

We showed in the preceding paper,[5] taking the three breakdown models proposed by Stowell et al.[14] into consideration, that the tensile strength σ_{cs} of a composite in which short fibers are oriented random-planar depends in a large measure upon the shear strength τ at the interface and is written as

$$\begin{aligned} \sigma_{cs} &\fallingdotseq \frac{2\tau}{\pi}\left[2 + \ln\left\{\frac{(1 - l_c/2L)\sigma_f\sigma_m V_f + \sigma_m\sigma'_m V_m}{\tau^2}\right\}\right] \qquad L \geq l_c \\ &\fallingdotseq \frac{2\tau}{\pi}\left[2 + \ln\left\{\frac{\tau(L/d)\sigma_m V_f + \sigma_m^2 V_m}{\tau^2}\right\}\right] \qquad L < l_c \end{aligned} \tag{12}$$

where L is the fiber length, d is the fiber diameter, l_c is the critical fiber length, σ_f is the tensile strength of the fiber, σ_m is the tensile strength of the matrix, V_f is the fiber volume fraction, V_m is the matrix volume fraction, and σ'_m is the matrix stress at the fracture strain of the fiber.

For the glass fiber–resin system as used in this experiment, there is the large difference in the thermal expansion coefficient; therefore, considerable thermal stress is produced during molding. Accordingly, actual strength of the composites is lower by the thermal stress than the theoretical strength given by eq. (12). The thermal stress σ_r is expressed as

$$\sigma_r = \frac{2(\alpha_m - \alpha_f)E_m\Delta T}{(1 + \nu_m) + (1 + \nu_f)(E_m/E_f)} \tag{13}$$

where α is the thermal expansion coefficient, E is Young's modulus, ν is the Poisson ratio, ΔT is the difference in temperature from the molding temperature, and the subscripts m and f represent matrix and fiber, respectively.

The theoretical strength $[\sigma_{cs}]_T$ of a composite in which short fibers are oriented random-planar can be given as below from eqs. (12) and (13), taking the effect of thermal stress into consideration:

$$[\sigma_{cs}]_T = \sigma_{cs} - \sigma_r \tag{14}$$

The solid lines in Figures 8 and 9 represent theoretical values of $[\sigma_{cs}]_T$ obtained by using eqs. (12), (13), and (14). In this theoretical calculations we employed the mean critical fiber length $\bar{l}_c$, Figures 7(a) and (b), for l_c, the mean fiber length $\bar{L}$ (Table III) for L, and the mean strength $\bar{\sigma}_f$ obtained by substituting the Weibull parameters σ_0, σ_p, m, and the number of links n (Table I) into eq. (4) for σ_f. The values of σ_m, σ'_m, and σ_r are shown in Table IV.

With these composites, the experimental values agree well with the theoretical values obtained by taking the effect of the thermal stress into consideration. It is therefore possible to estimate the tensile strength of composites reinforced with random-planar orientation of short fibers using eqs. (12), (13), and (14) over the entire range of experimental aspect ratios L/d 55–400 (fiber length 0.75–5.00 mm). Also the calculated values show better agreement with the experimental values than the values (dotted lines in Figs. 8 and 9) calculated by using the τ and l_c obtained on the assumption that the strength of the fiber is uniform as described in our preceding papers.[4,5] The fact that the calculated values, obtained by using the shear strength at the interface obtained by taking the distribution of fiber strength into account for τ which is the most important factor

TABLE IV
Strength and Stress of Matrix and Thermal Stress

Temp	40°C			80°C		
Matrix	Strength σ_m, ($\times 10^8$ dyn/cm^2)	Stress σ'_m, ($\times 10^8$ dyn/cm^2)	Thermal stress σ_r, ($\times 10^8$ dyn/cm^2)	Strength, σ_m, ($\times 10^8$ dyn/cm^2)	Stress σ'_m, ($\times 10^8$ dyn/cm^2)	Thermal stress σ_r, ($\times 10^8$ dyn/cm^2)
Epoxy	5.59	3.44	1.48	0.53	0.13	0.025
Unsaturated polyester	2.13	2.12	1.28	0.50	0.10	0.013

of eq. (12), agree better with the experimental values supports the reliability of the measured values of τ in the preceding section. Accordingly, consideration of distribution of fiber strength is important in measuring τ and l_c of glass fiber–resin systems according to the embedded method of this article.

As observed in Figures 8 and 9, the tensile strength is almost unchanged after the aspect ratio has reached a certain level. We will now discuss the effect of length of the fiber on the reinforcement effect in comparison with critical fiber length. As a basis of comparison, a theoretical tensile strength $\sigma_{c\infty}$ of composite reinforced with random-planar orientation of continuous fibers with the same volume fraction is obtained by changing $L \to \infty$ in eq. (12) and is written as

$$\sigma_{c\infty} \fallingdotseq \frac{2\tau}{\pi}\left[2 + \ln\left(\frac{\sigma_f \sigma_m V_f + \sigma_m \sigma'_m V_m}{\tau^2}\right)\right] \tag{15}$$

The theoretical tensile strength $[\sigma_{c\infty}]_T$ under consideration of thermal stress is given by substituting $\sigma_{c\infty}$ into eq. (14):

$$[\sigma_{c\infty}]_T = \sigma_{c\infty} - \sigma_r \tag{16}$$

We define the ratio of tensile strength of a composite reinforced with short fibers at a given temperature to the tensile strength at the same temperature obtained from eq. (16) as efficiency factor of short-fiber reinforcement:

$$\psi = \frac{[\sigma_{cs}]_T}{[\sigma_{c\infty}]_T} \tag{17}$$

The relationship between the efficiency factors ψ and $L/\bar{l}_c$ (fiber length ÷ mean critical fiber length) for the epoxy resin system and the unsaturated polyester resin system is shown in Figures 10 and 11, respectively, at 40 and 80°C. Regardless of the kind of matrix, the nature of interfacial treatment, and the environmental temperature, the efficiency factor ψ increases with increasing $L/\bar{l}_c$. In any composite, when the fiber length L is equal to the mean critical fiber length $\bar{l}_c$ of the system, that is, when $L/\bar{l}_c = 1$, the efficiency factor ψ is approximately 85%; and when $L/\bar{l}_c = 1.5$ and 5, the efficiency factors ψ are approximately 90% and 97%, respectively.

Accordingly, in this experiment a tensile strength as high as approximately 90% of the theoretical tensile strength of composite reinforced with random-planar orientation continuous fibers is obtained if a composite is reinforced with random-planar orientation short fibers of 1.5 times the mean critical fiber length $\bar{l}_c$ regardless of the kind of matrix, the nature of interfacial treatment, and the

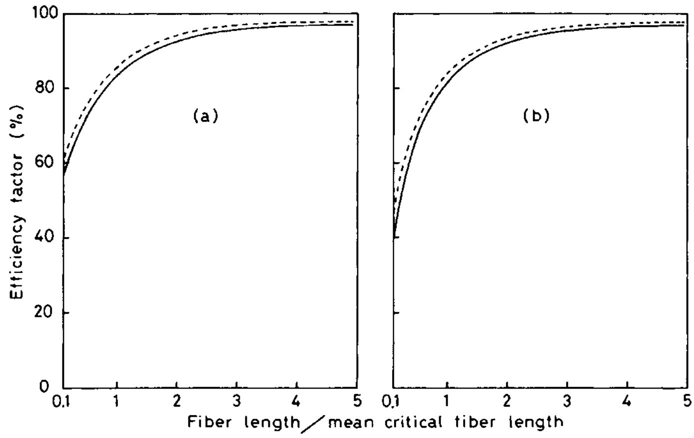

Fig. 10. Relation between fiber length/mean critical fiber length and efficiency factor (at 40°C): (a) epoxy–short glass fiber composite; (b) unsaturated polyester–short glass fiber composite; (—) good bonding; (- - - - -) poor bonding.

environmental temperature. Furthermore, a tensile strength as high as approximately 97% of theoretical strength is obtained when the length of the reinforcement fibers is 5 times the mean critical fiber length. However, in practice, the longer the reinforcement fiber length, the more difficult the molding process becomes. Therefore, it may be adequate to use fibers which will provide a strength of 90 to 95% of theoretical, i.e., short fibers of 1.5–2.0 times the mean critical fiber length.

CONCLUSIONS

A method was devised to accurately determine the shear strength at the interface, which plays an essential role in determining the reinforcing effect in discontinuous fiber-reinforced resins, taking the effect of the fiber strength

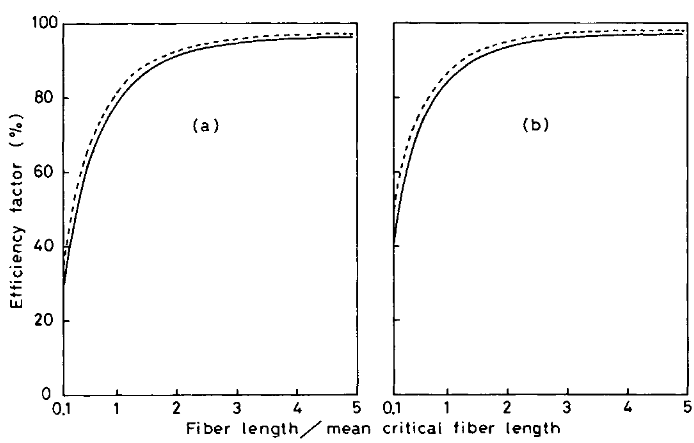

Fig. 11. Relation between fiber length/mean critical fiber length and efficiency factor (at 80°C): (a) epoxy–short glass fiber composite; (b) unsaturated polyester–short glass fiber composite; (—) good bonding; (- - - - -) poor bonding.

distribution into consideration. The theoretical tensile strength of composites calculated by employing the shear strength at the interface obtained by the method agrees better with the experimental ones than those calculated by employing the shear strength obtained on the assumption that the fiber strength is uniform.

The tensile strength of composites increases as the aspect ratio of the reinforcing fibers increases, and this trend is almost the same for both composites regardless of the kind of matrix, the nature of interfacial treatment, and the environmental temperature. A tensile strength of approximately 90% of the theoretical tensile strength of composite reinforced with random-planar orientation continuous fibers is obtained when a composite is reinforced with random-planar orientation short fibers of about 1.5 times the mean critical fiber length, and also approximately 97% with short fibers of 5 times the mean critical fiber length. However, taking practical processing facility into consideration, it will be most adequate to reinforce with short fibers of 1.5–2.0 times the mean critical fiber length.

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